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14. ABSTRACT

It would be interesting to envision a light, thin and heat resistant multifunctional material which when exposed to some kind of a blast wave at one side manages to rapidly absorb and redistribute the received energy into localized tiny hot spot regions between the layers thereby protecting a significant part of the blast from getting transmitted to the other end. Development of the scientific understanding needed for insuring that the perturbation does not become thermalized has been the focus of the proposed study. We believe that such research may find potential

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Report Title

On the Dynamics and Control of Mechanical Energy Propagation in Granular Systems

ABSTRACT

It would be interesting to envision a light, thin and heat resistant multifunctional material which when exposed to some kind of a blast wave at one side manages to rapidly absorb and redistribute the received energy into localized tiny hot spot regions between the layers thereby protecting a significant part of the blast from getting transmitted to the other end. Development of the scientific understanding needed for insuring that the perturbation does not become thermalized has been the focus of the proposed study. We believe that such research may find potential application in the development of novel impact absorption materials. The research shows that it is indeed possible to develop nonlinear oscillator systems that are capable of harnessing perturbations via multiple intrinsic localized modes. The studies also suggest that such energy redistribution may be possible to accomplish for a broad range of frequencies than what may have been thought to be possible in earlier pilot work.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received Paper

TOTAL:

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received Paper

TOTAL: **3**

Number of Papers published in non peer-reviewed journals:

(c) Presentations

American Physical Society March 2014 Meeting, Denver, CO
March 2-7, Session D-16

Lifetime and Decay of Breathers in the FPU System

Matthew Westley, Nicholas DeMeglio, Surajit Sen, T.R. Krishna Mohan, SUNY Buffalo

The Fermi-Pasta-Ulam problem [1] consists of a chain of N oscillators with linear and nonlinear nearest neighbor interactions. Using velocity-Verlet integration, we study the evolution of the system after a perturbation that consists of a single stretched bond at the center of the chain [2-4]. This perturbation results in the localization of most of the system's energy in the center particles in the form of a ``breather'' up to reasonably long times, which leaks energy at a rate depending on the potential parameters and the perturbation amplitude. The breather eventually undergoes a catastrophic breakdown, releasing all of its energy into acoustic noise and solitary waves. We explore the conditions on the amplitude and the parameters ?, ? for which a seeded breather will be most or least stable. Also we show how the overlap or lack thereof between the breather's primary frequencies and the acoustic frequencies influences its long-time stability. \\[4pt] [1] E. Fermi, J. Pasta, and S. Ulam, Los Alamos Scientific Laboratory Report No. LA-1940 (1955).\\[0pt] [2] S. Flach and A. V. Gorbach, Phys. Rep. \textbf{467}, 1 (2008).\\[0pt] [3] A. J. Sievers and S. Takeno, Phys. Rev. Lett. \textbf{61}, 970 (1988).\\[0pt] [4] T. K. Mohan and S. Sen, Pramana \textbf{77}, 975 (2011).

SIAM Meeting, SNowbird, Utah, May 2013 (Invited)

Long Lived Solitary waves in a 1D Granular Chain

Yoichi Takato and Surajit Sen, SUNY Buffalo

Kinetic energy fluctuations of a non-dissipative 1D granular chain held between reflecting walls with various precompressions are investigated. The dynamics is explored for a weakly precompressed chain which admits only solitary waves that break into secondary waves under wall collisions, for a strongly precompressed chain which exhibits acoustic waves, and for intermediate precompression. The last case accommodates a nearly stable solitary wave that travels unaffected through the acoustic waves for extremely long times

Forthcoming invited lecture on "Breathers in MEM systems" coming up in "The Propagation of Waves in Dissipative, Discrete Structures" that A Roasto and D Blackmore are organizing for the
13th International Symposium on Multiscale, Multifunctional and Functionally Graded Materials (MM&FGM 2014),
Oct. 19 - 22, 2014,Taua Resort, Sao Paulo, Brazil

Number of Presentations: 3.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received	Paper

TOTAL:

Number of Manuscripts:

Books

Received	Book

TOTAL:

Received Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

Elected Fellow, American Association for the Advancement of Science, December 2012

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Matthew Westley	0.35	
William J Falls	0.25	
Yoichi Takato	0.30	
Rahul Kashyap	0.10	
FTE Equivalent:	**1.00**	
Total Number:	**4**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Under Graduate students supported

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Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

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Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME
Total Number:

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Please see attachment

Technology Transfer

On the Dynamics and Control of Mechanical Energy Propagation in Granular Systems

Surajit Sen

Department of Physics, SUNY Buffalo

March 2014

Table of Contents

1 Objectives and Significance

Protection of humans, equipment and machinery from large amplitude, noisy, finite time perturbations is an important area of basic and applied research that has been pursued for many decades. It is hence of significant interest to learn about the underlying mechanisms that may someday allow us to develop thin, strong and light mass materials for purposes of protection from impacts [1]. Can we then make materials which would be very thin, very light, non-brittle and at the same time capable of damping sudden large changes in acceleration? While developing such materials has been a challenging journey, woodpeckers provide an example of what may someday be possible! Typically, woodpeckers peck on wood about 20-22 times per second. Such rapid impacts can result in acceleration changes that are ~ 1200g [3]. However, they don't get brain damage! An acceleration change of 1200g is a good order of magnitude larger than the protection that the best helmets can provide [4, 5]. Studies show that the woodpecker skulls are thin and lightweight and have almost no bulk fluids (**Fig. 1**) [3, 6, 7]. Their skulls also have a dense plate-like structure and have low porosity [2]. How exactly these skulls absorb such large acceleration changes is presently unclear.

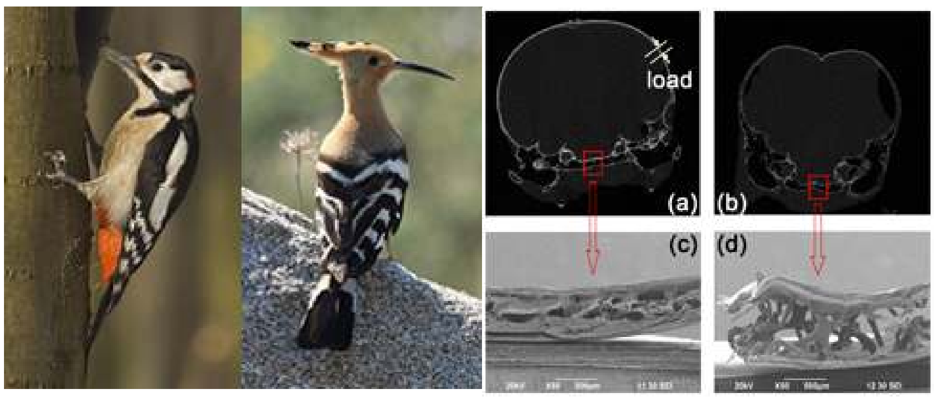


Fig. 1(left): Great spotted woodpecker on the left and the Eurasian Hoopoe on the right; **Fig. 1(right):** Woodpecker's head and skull bones on the left (a,c) and Hoopoe's head and skull bone on the right (b,d) (from Ref [2]).

Of course, one way to develop such materials would be to mimic the woodpecker skull itself. But that is not easy to do. Here we choose to pursue a somewhat different path, an "out-of-the-box" ***idea, of capturing the incoming energy in pockets all over a dense network of tiny micro-electro-mechanical (MEM) cantilevers*** [1, 8, 9]. If a single cantilever can be made to temporarily absorb a good fraction of the incoming energy then one can imagine each cantilever as an "edge" or a bond, where each bond is a link in a network (see **Fig. 2**), much like in bones. The idea of using MEM cantilevers as possible impact absorbers builds on the rapid developments that have transpired in nonlinear dynamics over last some sixty years or so [10, 11]. The use of MEM cantilevers for possibly capturing incoming energy originates from the experimental and simulational studies by Sievers and coworkers [12]. Will the idea work? In the following sections we build a plan to explore that question. If the idea eventually works out, one may someday be able to make very light, small, impact absorbing materials which can be very useful.

As we shall discuss in some detail below, the presence of strongly nonlinear forces in many particle systems can result in unexpected ways of energy transport in the system [13]. In other words, in strongly nonlinear systems, energy may not quite disperse as it would in a chain of harmonic oscillators [10, 13-15].

These systems typically allow the existence of solitary waves, and anti-solitary waves, which are uniformly propagating, non-dispersive compression and dilation pulses, respectively [16-23]. In addition, these systems admit intrinsic localized modes (ILMs) or breathers [8, 12, 15-18, 20, 22, 24-50], which are typically regions in the system where energy can get temporarily trapped, sometimes for rather long times compared to typical system relaxation time scales. It is this energy localization property, the one associated with the ILMs or breathers that seem to be especially germane in the context of harnessing incoming energy in parts of a system such as in **Fig.2**. If the breather can be harnessed for a sufficiently long time, the energy can slowly dissipate via formation of hot regions or via some local damage. The energy can also presumably drain through some filler material that can be placed in the porous space of the network in such a way that very little energy penetrates the entire network, thus serving our purpose.

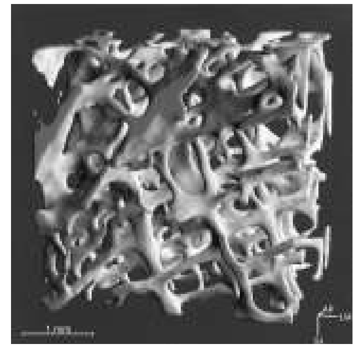


Fig. 2: A generic bone network is shown. This picture is taken from the following URL - http://www.lencoheaven.net/forum/index.php?topic=11143.0

The broad based basic research question to address then is ***whether the MEM cantilevers can be designed to rapidly capture large amplitude noisy signals into one or more highly stable energetic breathers***. Experimental and simulational studies carried out by Sievers and coworkers [1, 8, 9, 38, 51-56] suggest that a fairly narrow distribution of driving frequencies across the right frequency window effected across finite times can indeed precipitate multiple breathers in MEM cantilever systems [1, 38, 53, 55]. Preliminary studies completed by us also confirm these findings and go beyond. Indeed the studies suggest that it would be promising to explore whether some form of MEM cantilever systems can actually be tuned up to trap breathers when subjected to random perturbations. ***One can then ask whether and how such breathers can be effected in various MEM cantilevers of a network of cantilevers such as shown in Fig. 2. The completed study is described in Sec. 3.***

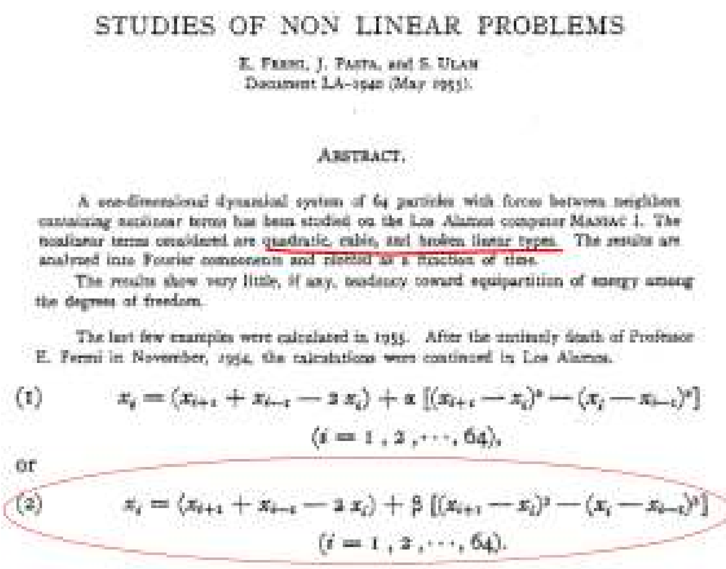
2 Background

The study of breathers began with the early works of Sievers and coworkers in the mid 1960s [12, 24, 57-60] and of others such as Maki and Takayama [40], Ovchinnikov [14], Stoll, Schneider and Bishop [15], and of Bishop [41, 61-66], Campbell and coworkers [11, 41, 61, 67] and others through the 70s. It appears that extensive analyses of breathers or ILMs by a great many research groups started with the works of Flytzanis, Pvenmatikos and Remoissenet [16], Sievers and Sato and Sievers and coworkers [1, 8, 9, 38, 51-56, 68]. Since the 1990s, a vast body of literature has emerged with investigations under way in a great many distinguished groups around the world. Since the community of researchers comes from mathematics, physics, and engineering, a variety of different kinds of studies are found, which has made the exploration of breathers rich and interesting. But to get a better sense of what kind of objects these breathers are, perhaps it is good to first back up a bit and trace out the threads that caught the attention of so many scientist on long lived excitations in

Fig. 3: Masses connected by springs being given a simultaneous compression and dilation at time t = 0. Figure taken from http://www.scholarpedia.org/article/Fermi-Pasta-Ulam_nonlinear_lattice_oscillations.

nonlinear systems in the first place.

Studies of nonlinear many body systems began in earnest with the studies of Fermi, Pasta and Ulam (**Fig. 3,4**) and the scientist who did the computations they envisioned, Betty Tsingou, in the Los Alamos MANIAC-1 (Mathematical analyzer, numerical integrator and computer) in 1955. They found that the ***presence of nonlinear spring like forces in a mass-spring chain held between rigid boundaries can result in a lack of thermalization of the system***.


STUDIES OF NON LINEAR PROBLEMS

E. FERMI, J. PASTA, and S. ULAM
Document LA-1940 (May 1955).

ABSTRACT.

A one-dimensional dynamical system of 64 particles with forces between neighbors containing nonlinear terms has been studied on the Los Alamos computer MANIAC I. The nonlinear terms considered are quadratic, cubic, and broken linear types. The results are analyzed into Fourier components and plotted as a function of time.

The results show very little, if any, tendency toward equipartition of energy among the degrees of freedom.

The last few examples were calculated in 1955. After the untimely death of Professor E. Fermi in November, 1954, the calculations were continued in Los Alamos.

$$x_i = (x_{i+1} + x_{i-1} - 2x_i) + \alpha[(x_{i+1} - x_i)^2 - (x_i - x_{i-1})^2] \quad (i = 1, 2, \cdots, 64), \tag{1}$$

or

$$x_i = (x_{i+1} + x_{i-1} - 2x_i) + \beta[(x_{i+1} - x_i)^3 - (x_i - x_{i-1})^3] \quad (i = 1, 2, \cdots, 64). \tag{2}$$

Fig 4: The above snapshot is from the original unpublished report of FPUT (whose name appears later). We discuss the system described by Eq. (2) on the page. Note the right hand side of Eqs (1) and (2) are actually second derivatives of x_i

Later, in 1965, Zabusky and Kruskal [13, 69] analyzed the Fermi, Pasta, Ulam and Tsingou (FPUT) system in the continuum limit via the Korteweg-deVries equation [69] and showed that these systems support solitary waves, which are propagating, non-dispersive bundles of energy. The Korteweg deVries equation, a nonlinear equation that is known to describe solitary waves in narrow, shallow and long channels, was solved in 1895.

Fig.5: Slide showing the names and years in which historic advances were made in understanding nonlinear waves

It should be mentioned here that the study of nonlinear equations may have originated with the desire to understand the origins of rogue waves which were perceived to be dangerous during the great ocean voyages starting in the Late Middle Ages (13th century) [68]. Some of the most well-known mathematicians and physicists in the history of science, engineering and mathematics participated in the studies that eventually led up to the works of Korteweg and de Vries and of others (see **Fig. 5**) [69]. Hence, large amplitude fluctuations (presumably rogue waves), traveling non-dispersive energy bundles (solitary and anti-solitary waves) and localized energy pockets (breathers) and a surprisingly rich mix of interactions of these objects are encountered in many strongly nonlinear systems. These objects hence seem to show up in pretty much all the nonlinear equations that describe discrete many body or continuum systems.

Sievers and Takeno's 1988 [12] work along with the preceding studies established that localized high frequency modes that are above the phonon frequencies and separated by a gap can exist. These vibrations are the intrinsic localized modes (ILMs) or breathers. Breathers can arise in the presence of very small proportions of impurities and at finite temperatures. Alternately, breathers are seen in pristine mass distributions and in three dimensional lattices without impurities (such as NaI) and in biological molecules [70].

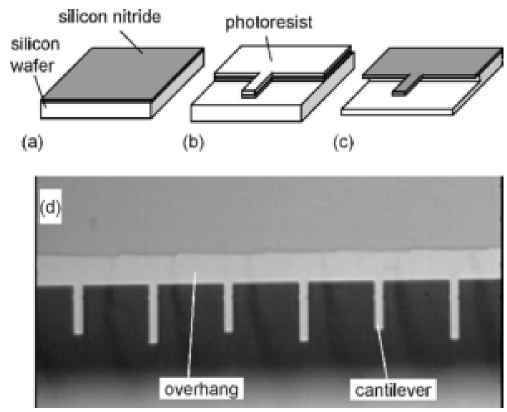


Fig. 6: The MEM cantilevers with two different overhang lengths relating to a diatomic lattice is shown (taken from Sato et al, [1]

Clearly, the whole idea of having some localized and long lived excitations among lattice vibrations raises intriguing questions that may require many years to answer. Let us not forget the FPUT system is already 59 years old and is the starting point of modern nonlinear many body work [71]. To achieve our present goal, i.e., to understand under what conditions energetic breathers can form and can stay localized for desired times, we need to broadly understand how the relevant system evolves for various forms of initial and boundary conditions and for various values of the parameters. We will present a brief overview of the past efforts below.

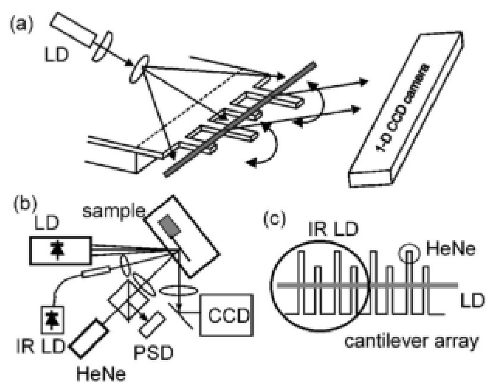


Fig.7: Taken from Sato et al.[1], the sample is driven by a piezo-electric transducer and the excitations are probed by a HeNe laser. The image from a highly excited cantilever will most likely miss the CCD camera and produce a dark spot.

The works of MacKay [35, 36, 44, 49, 72-76], Malomed [22, 23, 77-91], Flach [11, 47, 49, 50, 92-103], Rosenau [30, 45], Kivshar [11, 18, 19, 29, 30, 67, 104-111], Peyrard [36, 37, 43, 47, 77, 112-117], Dauxois [42, 43, 112, 113, 115, 118], Ruffo [118, 119], Willis [50, 92, 93, 112, 113], Aubry [35, 44, 46, 72, 101, 120-127], Kiselev [27, 31, 70, 128-130], Page [28, 32, 131], Livi [74, 132], Cretegny [95, 118, 124, 126, 133], James [134, 135], Lindenberg [136], Kevrekidis [117, 133, 137], Konotop [138-140], and their coworkers along with the works of Sievers, Takeno and Sato [1, 8, 9, 38, 51-56, 68], Bishop [41, 61-66], Campbell [11, 41, 61, 67, 137], Porter [137], Daraio and coworkers [137] and others continue to shape the landscape of understanding of breathers in the broader context for discrete and continuum systems since the 1990s or so. The focus on the MEM cantilever systems began around 2003. This system may be particularly important for the present study (see **Fig. 6,7**). The works of Sato, Hubbard, Sievers and several others have shaped our present understanding of the MEM cantilever system.

The system in **Figs. 6,7** is set up in such a way that the cantilevers oscillate perpendicular to the plane of the MEM cantilever system. However, it is possible to show that effectively the dynamics is effectively 1 dimensional (hence the equation used by Sato et al in **Fig. 8**). The masses (~10^{-13} kg) interact with each other *effectively* via a quadratic and a quartic potential. The masses also experience inter-site quadratic and quartic potentials as well as on-site quadratic and quartic potentials. The quartic interactions are significantly stronger in both on-site and inter-site cases. We will refer to this system henceforth as the Sievers system. The experimental set up (**Fig. 7**) describes the way in which the system is initiated. The system is driven via a driver through a range of frequencies where it is likely to pick up oscillations at frequencies above the phonon band or in a gap. The right frequencies and driving along with specific excitations across selected windows can precipitate breathers. The breathers once formed would be unable to relax easily by coupling with the phonon band. And ***it is the nature of the couplings that would be important in deciding the lifetime of the breather.*** Of course the experimental system has dissipation. Eventually all energy would die out.[1]

$$\begin{aligned} m_i \frac{d^2x_i}{dt} &+ \frac{m_i}{\tau}\frac{dx_i}{dt} + k_{2Oi}x_i + k_{4Oi}x_i^3 \\ &+ \sum_j k_{2I}^{(j)}(2x_i - x_{i+j} - x_{i-j}) \\ &+ k_{4I}[(x_i - x_{i+1})^3 + (x_i - x_{i-1})^3] \\ &= m_i \alpha \cos(\Omega t), \end{aligned}$$

Fig.8: The effective nonlinear equation experimentally and simulationally probed by Sato et al. [1]

Sato and coworkers did both experiments and simulations to see the emergence of the breathers upon chirping the otherwise driven system in a window around the optic branch (~140kHz) frequency for some 16 ms (**Figs. 9, 10**). The driving itself continued for times which are nearly 3 times longer. The system showed that the driving precipitated breathers. Some of these breathers were strong and lasted for some 20ms or so (a rather long time here). A snapshot of the experimental and simulational data of Sato et al is

Type	Hard di-element lattice	Hard di-element lattice	Soft mono-element lattice
m (10^{-13} kg)	7.67 (6.98)	7.67 (6.98)	7.67
τ (ms)	8.75	8.75	8.75
k_{2O} (kg/s^2)	0.102 (0.0976)	0.142 (0.168)	0.142
k_{2I1}–k_{2I6} (kg/s^2)	0.104, 0.0405, 0.0189, 0.0118, 0.00887, 0.00346	0.0828, 0.0308, 0.0108, 0.00405, 0.00250, 0.000824	0.0828, 0.0308, 0.0108, 0.00405, 0.00250, 0.000824
k_{4O} (kg/s^2 m^2)	1.00×10^8	1.00×10^8	1.00×10^8
k_{4I1}–k_{4I6} ($\times10^{10}$ kg/s^2 m^2)	4.0, 0,0,0,0,0	4.0, 0,0,0,0,0	4.00, 1.49, 0.522, 0.195, 0.120, 0.040
S (μm^2)			55×15
d (μm)			10
V (V)			40
f_d (kHz)	147–151	139	64.6
α (m/s^2)	14000	5000	290
f_1, f_2, f_3, f_4 (kHz)	58.78, 133.3, 139.8, 147.4	73.04, 123.4, 133.0, 137.1	65.67, 130.5

Fig 9: The parameters used in studying the equations in **Fig. 7** are given above. The Table is from Sato et al. [1]. A little introspection shows that the natural units to describe this system is nanograms, microns and microseconds

[1] The author acknowledges helpful discussions with Prof A.J. Sievers at various stages of his study on breathers.

shown in Ref. [1]. It is also known that frequencies seen in nonlinear systems are strongly dependent on perturbation amplitudes and hence perturbation energies. Thus, if one understands enough about how to design and control the dynamics of these types of systems (may not be exactly the same systems as those of Sievers et al.), then it may be possible to actually trap breathers from noisy large amplitude perturbations and hold them long enough for them to decay before they disperse. This realization inspires us to propose the study below. While the idea may seem rather far-fetched, our pilot work suggests a strong chance of success.

3 Research Accomplished

3.1. The systems:

Clearly, considerable attention has been paid to how breathers form in the FPUT system, the Sievers system and other systems. We are interested in breathers for the purposes of making light, small, and highly efficient impact absorption systems. To this end, ***the overall objective here is to determine the following in a single 1D system. What range of values of α and β, for what kinds of perturbation amplitudes over time and for what mass distributions will be best for generating multiple stable and energetic breathers?***

We would also like to understand why a set of parameter ranges would work. And in the future to be able to go deeper into the equations to get the big picture of the dynamics as functions of the 5 parameters stated above. By "energetic" we mean one or more breathers that carry roughly ~ 60+% or so of the total energy imparted to initiate motion in the system.

The equation of motion for any particle in the FPUT system and the same for the Sievers' system differ due to the presence of the on-site terms. These terms are present only in the Sievers system. The next important point to note is that the Sievers system appears to have extremely strong quartic interactions at the on-site and inter-site terms compared to the same in the corresponding quadratic terms (see Table in **Fig. 9**). However, these interaction strengths have thus far been seen in standard SI units. For a system of size ~ 10^2 microns, total mass ~10^{-9} gms, with excitations in the kHz regime (see **Fig. 9**), SI units may not be the most useful to develop some insights into the dynamics. If one looks at the Sievers system using nanogram, micron and microseconds as the units of mass, length and time, respectively, the quadratic on-site and inter-site couplings are about the same strength. The quadratic inter-site coupling turns out to be an order of magnitude stronger than the quartic inter-site coupling. The quadratic on-site coupling works out to three orders of magnitude stronger than the quartic on-site term. Thus, in these "natural" units, the quartic couplings look relatively weak in the Sievers system. ***Our studies suggest that the parameter ranges accessed in the Sievers studies may not be the best window to search for parameters that would serve our needs and this observation. Thus a broad based understanding of breather formation is needed.***

A second important factor is the amplitude of the perturbation that initiates the system. If this is weak compared to the amplitude of typical gentle vibrations of particles, then strong nonlinear forces may not kick in early and their effects may be evident only after carefully tuned driving at late times. However, if the perturbations are somehow strong, even with weak quartic couplings, strongly nonlinear effects may enter early. However, how much of linearity and nonlinearity are needed is not clear and this needs to be sorted out in detail. For now we will ignore dissipation though we have done pilot work on driven dissipative versions of these systems. We will take dissipation into account later and this turns out to be straightforward to do in dynamical simulations. ***Below we describe the pilot level work we have done and then describe the specific task items we will pursue to accomplish our objectives.***

> ***Task 1: A model system with its corresponding Newtonian dynamical equations will be set up to describe a one dimensional chain of quadratic + quartic oscillators which are connected by harmonic springs and then the system will be mapped to a corresponding MEM cantilever array in the manner shown by Sievers and coworkers (2003).***

Though in the experiments on the Sievers' system, the driven dissipative configuration precipitated multiple breathers, our results suggested that the driving may have been the crucial factor in precipitation of the breathers. The Sievers's simulations do not provide any details and hence we were unable to specifically recover his simulations though our work is in overall agreement with those of Sievers and coworkers.

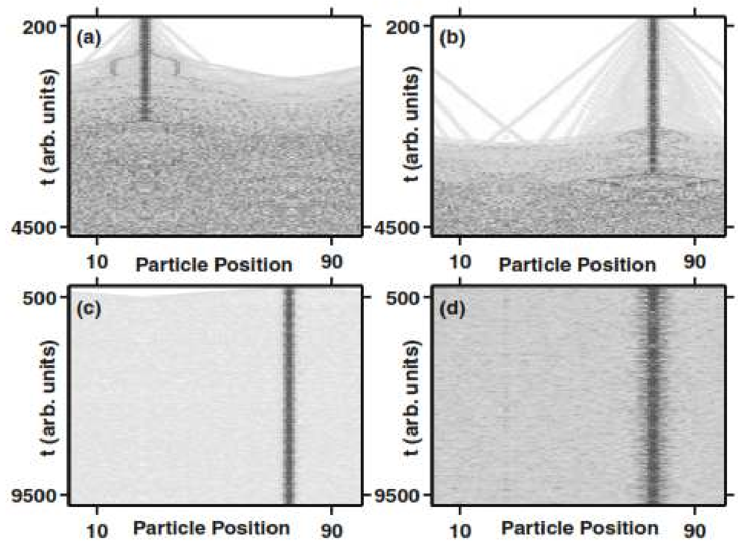


Fig.10: (a), (b), (c) and (d) show breathers in the FPUT system for $\alpha = 0, \beta = 1$, $\alpha = 10^{-2}, \beta = 1$, $\alpha = 10^{-1}, \beta = 1$ and $\alpha = 1, \beta = 1$, respectively. Dark line shows increased energy

Our study thus far has been broad based. We focused on understanding the dynamical properties of the breathers in these systems. If one understands what makes them metastable and unstable, it should be possible to construct the right kind of systems and conditions to realize energetic breathers that we would like to make for our purposes. With this in mind we considered monodispersed chains with $N = 100$ masses and with the end masses being connected to infinitely heavy walls (**Fig. 10**). We considered two kinds of scenarios for both the FPUT and the Sievers systems – ***seeded breathers and unseeded breathers***. Our studies on each of these are addressed below.

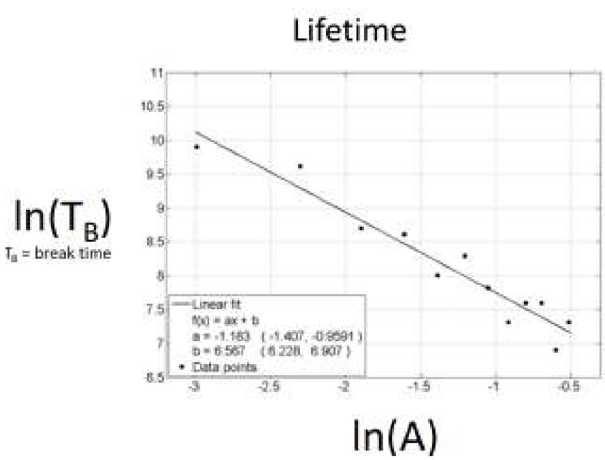
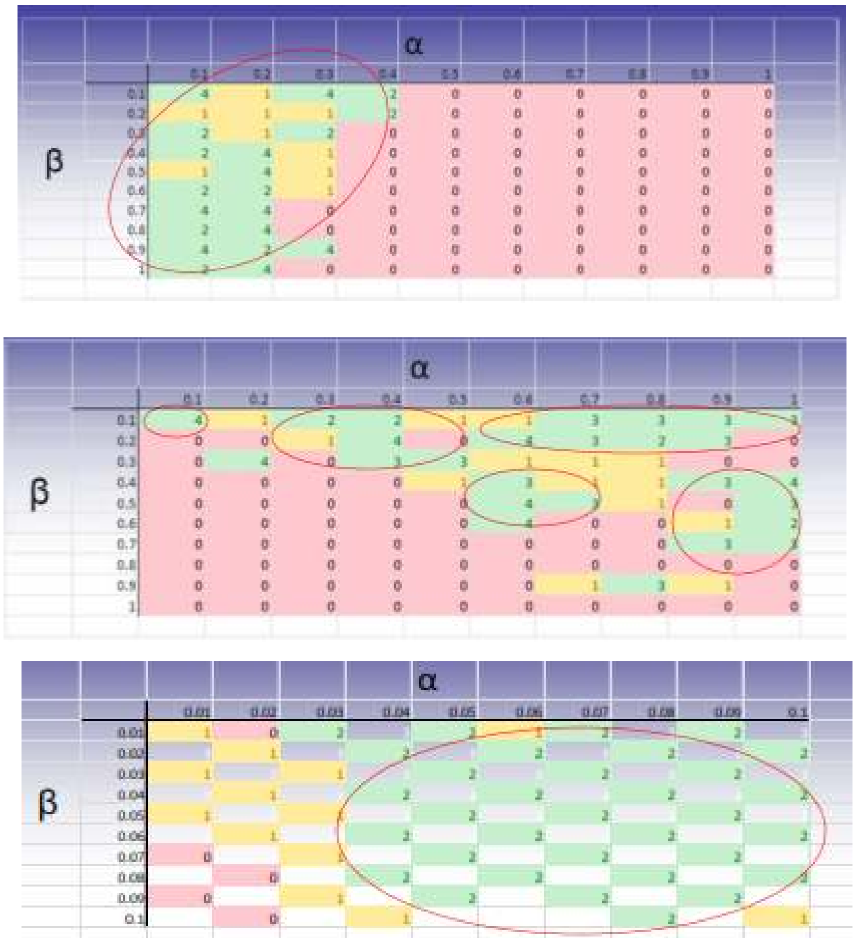


Fig.11: Dependence of breather lifetime on perturbation amplitude for $\alpha = 0, \beta = 1$.

We will first talk about ***seeded breathers in the FPUT systems***. In studying ***seeded breathers***, at time $t = 0$, a breather was initiated by stretching a bond at the center of the chain. In studying ***unseeded breathers***, we subjected the equal masses in the system to random perturbations at $t = 0$. Seeding always produced breathers that were highly stable for a significant amount of time (**Fig. 11**), with breather lifetimes becoming large enough to become intractable in computational studies when $\alpha > 0.3, \beta = 1$ and for sufficiently small perturbation amplitudes. Further the breathers carried about 95% of the total system energy until breakdown after many decades in time – hence these were the strong and energetic breathers we were looking for. ***Clearly for device purposes, it would be desirable to have systems in which perturbations can somehow seed breathers perhaps by straining the systems initially and letting the strains be released by the impact***. As for ***breathers in unseeded systems***, they were much harder to generate out of random perturbations in uniform mass systems. The studies we have done for the FPUT problem suggest that there are windows of parameter values for which breathers spontaneously form out of random perturbations initiated at $t = 0$ and without any extended driving (**Fig. 12**). This is what we want for impact absorption purposes. Impulses seldom behave like protracted driven signals at fixed frequencies. It has been difficult to precipitate breathers in the Sievers system from such random initial perturbations, however.



β \ α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	4	1	4	2	0	0	0	0	0	0
0.2	1	1	1	2	0	0	0	0	0	0
0.3	2	1	2	0	0	0	0	0	0	0
0.4	2	4	1	0	0	0	0	0	0	0
0.5	1	4	1	0	0	0	0	0	0	0
0.6	2	2	1	0	0	0	0	0	0	0
0.7	4	4	0	0	0	0	0	0	0	0
0.8	2	4	0	0	0	0	0	0	0	0
0.9	4	2	4	0	0	0	0	0	0	0
1	2	4	0	0	0	0	0	0	0	0

β \ α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	4	1	2	2	1	1	3	3	3	3
0.2	0	0	1	4	0	4	3	2	3	0
0.3	0	4	0	3	3	1	1	1	0	0
0.4	0	0	0	0	1	3	1	1	3	4
0.5	0	0	0	0	0	4	3	1	0	3
0.6	0	0	0	0	0	4	0	0	1	2
0.7	0	0	0	0	0	0	0	0	3	3
0.8	0	0	0	0	0	0	0	0	0	0
0.9	0	0	0	0	0	0	1	3	1	0
1	0	0	0	0	0	0	0	0	0	0

β \ α	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.01	1	0	2		2	1	2		2	
0.02		1		2		2		2		2
0.03	1		1		2		2		2	
0.04		1		2		2		2		2
0.05	1		1		2		2		2	
0.06		1		2		2		2		2
0.07	0		1		2		2		2	
0.08		0		2		2		2		2
0.09	0		1		2		2		2	
0.1		0		1				2		1

Fig.12: (upper) Circled green regions show breather formation in an FPUT system when it is perturbed at $t = 0$ with random spring displacements (amplitude $A = \pm 0.2$, 1 being the bond length). All masses are the same; (middle) region where breathers emerge when random velocity perturbations are given to a monodispersed system; (c) breather formation when masses are uniformly randomly distributed between two limits, 0.5 and 1.5 and weak velocity perturbation with initial velocities between +0.1 and -0.1 are used. Yellow regions contain solitary waves and the pink regions are very noisy quasi-equilibrium phases.

Breathers precipitated from random perturbations have typically turned out to carry less than 15% of the energy, and hence are not quite acceptable for highly effective impact absorption applications.

Task 2: The bimodal and related systems with two or three different masses in each unit cell will be perturbed at every site using random velocities at one initial instant and then through a time interval. These nonlinear chains have not been exhaustively probed to our knowledge using such broadband excitations. The dynamics of the system will be probed using the velocity Verlet algorithm and with dissipation. The principal objective would be to determine the conditions under which such broadband excitations can precipitate intrinsic localized modes.

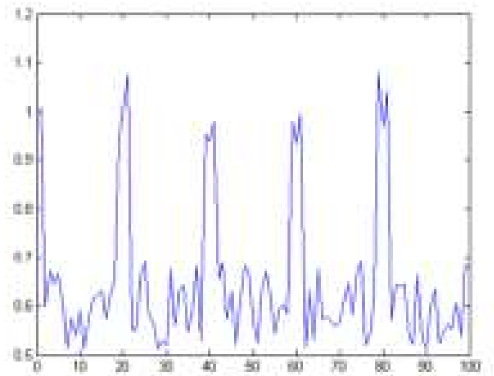


Fig.13: A typical random distribution of masses shown in a Sievers system study.

We have done extensive work on the Sievers system with randomly distributed masses between two limits (**Fig. 13**). Hence, these systems are disordered bimodal systems. Work done so far strongly indicates that bimodal or random mass systems are capable of precipitating moderately strong breathers starting from random perturbations at $t = 0$ (**Fig. 14**). We find that it is indeed possible to precipitate strong breathers in impurity sites in these systems. Our studies have been done for systems which have been preferentially driven at certain frequency windows as done by Sievers et al. We have studied these systems both without dissipation and with dissipation.

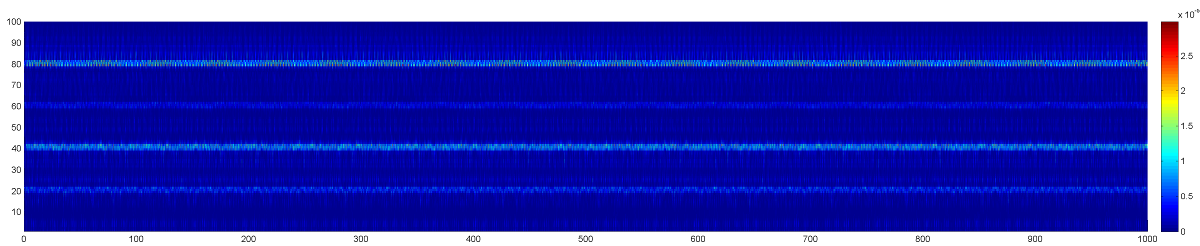


Fig.14: Here we show the formation of extremely stable breathers in a driven Sievers system with the mass distribution shown in Fig.11. Sievers' parameters for a diatomic chain are otherwise used. However, we would like to explore systems where breathers are initiated by just about any form of perturbation and not such specific kinds of perturbations.

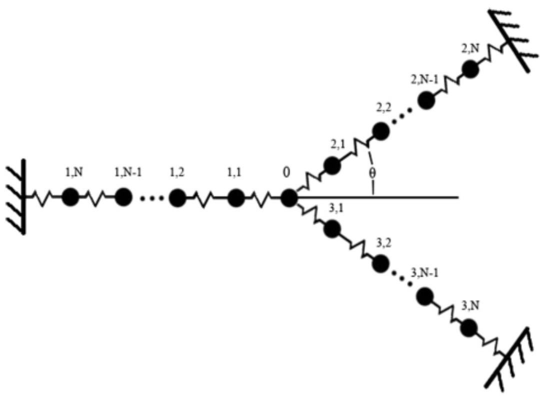
Task 3: Finally, step (i) will comprise of a check on the robustness of the dynamics for different inertial mismatches. In this stage, the inertial mismatches between the two cantilevers in a unit cell (see Fig 1) will be varied and also the role of small fluctuations in these masses will be explored.

Due to all the challenges associated with the problem of precipitating breathers, we deemed it premature to try to make breathers in a system of two or more cantilever units. However, we have carried out studies to see how mechanical energy is transmitted through a "Y" shaped structure by initiating a velocity perturbation at a point in an FPUT chain and seeing how this perturbation propagates to different branches. The studies were done for strongly nonlinear cases and for strongly linear cases (**Fig. 15**). Our studies show that the strongly nonlinear "Y" shaped structures and trees made of iterations of such "Y" shaped structures are capable of better transmission of energy from the stem to the breathers created at each of these structures interact have not yet been probed (**Fig. 16**).

$$H = \sum_{a=1}^{3}\left(\sum_{i=1}^{N_a} \frac{p_i^2}{2m} + V_{i,i+1}\right) + \frac{p_0^2}{2m} + V_{0,11} + V_{0,21} + V_{0,31}$$

$$V_{i,i+1} = \frac{1}{2}k(x_{i+1} - x_i - d_0)^2 + \frac{1}{4}q(x_{i+1} - x_i - d_0)^4$$

Fig.15: Here we show the Hamiltonian used for setting up a "y" shaped structure (see below). Each branch is "on-rails" with a vertex particle linking the rails. Every particle is linked by an FPUT potential.



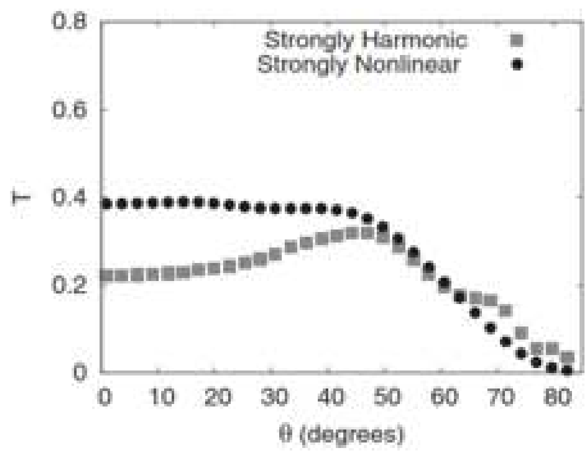


Fig.16: (Left) A system of three FPUT chains that are linked with a vertex particle defines the "Y" shaped system. This system is the building block for making networks of cantilevers. (Right) Here mechanical energy transmission T is shown where T is proportion of the energy in the horizontal stem that passes on to an adjacent branch. Cases where the potential is strongly harmonic and where it is strongly nonlinear have been probed. The results clearly show that non-linear potentials allow transmission of more energy when the angle θ is not very large (see Figs).

Additional/related work

Following additional/related studies have been carried out in order to develop broad insights into the FPUT and Sievers problems.

1. The PI has carried out an analyses of the nature of large energy fluctuations in these systems and linked them with the well-known studies on rogue waves in the deep ocean that are probed using the nonlinear Schrodinger equation.

2. The PI has carried out a detailed study of how a nonlinear system of the FPU type evolves when two ends of the system have different dynamical properties. The study shows that under certain conditions, at late enough times ergodicity is achieved.

3. The PI has led a study in which interactions between colliding nanoparticles have been studied. Though apparently unrelated, this study was designed to understand whether interactions between particles in the smallest scales are profoundly affected by the size itself.

4. Next Steps

We are looking forward to continued progress in using dynamical simulations as a guide to future experiments on how to trap incoming energy as breathers in FPUT and Sievers type systems. Our goal is to convert noisy perturbations into breathers. And we also want to seek out systems that may be potentially capable of converting low frequency and large amplitude perturbations into breathers. To this end, deeper exploration of the parameter space is needed. Further, studies on 2D and 3D network systems are needed and we look forward to doing such studies in the immediate future.

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